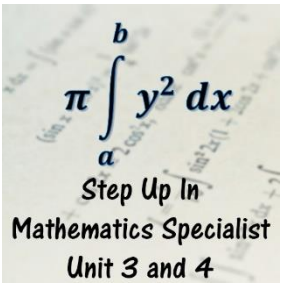
5.4 Application of Differential Equations

Problems Worksheet



1. Exponential growth.

    a. State the differential equation which represents exponential growth and define all terms.

    b. State the general solution to the DE in part a.

    c. Derive your answer in part b by solving the DE from part a.

    d. Demonstrate that your answer in part b is indeed the solution to the DE in part a.

2. Newton's Law of Cooling.

    a. State the differential equation Newton's Law of Cooling represents and define all terms.

    b. State the general solution to the DE in part a.

    c. Derive your answer in part b by solving the DE from part a.

    d. Demonstrate that your answer in part b is indeed the solution to the DE in part a.

3. Logistic equation.

    a. State the differential equation the logistic equation represents.

    b. State the general solution to the DE in part a.

    c. Derive your answer in part b by solving the DE from part a.

4. Using the general solution of the appropriate DE, complete the following:

    a. A small mining town is experiencing exponential population growth at a rate of 6% per annum. If its population on 1 January 2015, when this growth began, was 10 000, write an expression for the population of the town t years later and hence determine in which year the population will first exceed 13 000.

    b. A city with exponential population growth has a population of two million in 1968 and three million in 2018. Estimate the population in 2028 to the nearest thousand if this trend were to continue.

    c. A freshly baked cake is removed from the oven, which was set at $180\ ℃$, and placed in a room where the temperature is a constant $24\ ℃$. After ten minutes its temperature was $80\ ℃$. For how much longer must the cake rest before it can be eaten, if the chef is waiting for it to reach $40\ ℃$ before he serves it?

5. Using the general solution of the appropriate DE, complete the following.

    a. A new battle royale game is taking Australia's youth by storm! On 1 January 2017 there were only 10 000 players, however it was estimated that 30% of Australia's 6.1 million 10 to 29-year-olds will have become regular players after three years, and 130 000 people were known to play regularly as at 1 July 2017. Assuming a logistic model of growth, write an equation for the gaming population as a function of time and determine the month in which this population first exceeded one million.

    b. A group of scientists is trying to reintroduce native birds to an area that was previously known to support a maximum of 10 000 such birds. The group will consider their work a success if a colony of 9 000 can be re-established. Suppose 500 birds were initially reintroduced and one year later the population was estimated to be 850. Assuming a logistic model of growth, determine how long it will take before the colony will be successful.